RX-003-1016001

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Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March - 2019

Mathematics : Paper - 8

(Graph Theory & Complex Analysis - II)

(New Course)

Faculty Code : 003
Subject Code : 1016001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are **compulsory**.

(2) Figures on the right side indicate marks.

1 (a) Answer the following questions briefly : **4**

(1) Define : Simple graph.

(2) Write the Nullity of connected graph with n vertices and e edges.

(3) Find the number of pendant vertices in any binary tree with 13 vertices.

(4) Find the total number of edges in a complete graph with 5 vertices.

(b) Attempt any **one** : **2**

(1) Define : Null graph, Pendant vertex.

(2) What is the number of vertices in the complete graph K_n, if it has 45 edges ?

(c) Attempt any **one** : **3**

(1) State and prove first theorem of Graph theory.

(2) Prove that the number of vertices n is a binary tree is always odd.

(d) Attempt any **one** : **5**

(1) Prove that a simple graph with n vertices and K components can have atmost $\frac{(n-K)(n-K+1)}{2}$ edges.

(2) Prove that A graph is a tree iff it is minimally connected.

2 (a) Answer the following questions briefly : **4**

(1) Define : Separable Graph.

(2) Define : Chromatic number of graph.

(3) Kuratowski's second graph $K_{3,3}$ has ______ edges.

(4) Define : Acyclic digraphs.

(b) Attempt any **one** : **2**

(1) Find the number of edges of a connected planar graph with 4 vertices and 4 regions.

(2) Write the chromatic number of null graph and complete graph K_n.

(c) Attempt any **one** : **3**

(1) If G is a simple, connected planar graph with f regions, n vertices and e edges $(e > 2)$ then prove that

(i) $e \geq \frac{3}{2} f$

(ii) $e \leq 3n - 6$

(2) Prove that every tree with two or more vertices is 2-chromatic.

(d) Attempt any **one** : **5**

(1) If G is a graph with n vertices, e-edges, f-faces and K components then prove that $n - e + f = K + 1$.

(2) Define path matrix and state is properties.

3 (a) Answer the following questions briefly : **4**

(1) Define : Bilinear mapping.

(2) Write the critical points of bilinear transformation $W = \frac{az+b}{cz+d}$.

(3) Find fixed point of the bilinear transformation $W = \frac{3Z-4}{Z-1}$

(4) The points which coincide with their transformation are called ________.

(b) Attempt any **one** : **2**

(1) Show that $W = \dfrac{az+b}{cz+d}$ is conformal mapping.

(2) Show that $x + y = 2$ transform into the parabola $u^2 = -8(v-2)$ under the transformation $W = Z^2$.

(c) Attempt any **one** : **3**

(1) Prove that the transformation $W = 2Z + Z^2$ maps the unit circle $|Z| = 1$ of Z-plane into a cardoide in W-plane.

(2) Find the bilinear transformation which maps $Z_1 = \infty, Z_2 = i, Z_3 = 0$ onto $W_1 = 0, W_2 = i$ and $W_3 = \infty$.

(d) Attempt any **one** : **5**

(1) Show that the composition of two bilinear transformation is again a bilinear transformation.

(2) Prove that the transformation $W = \dfrac{1}{Z}$ maps the circle $|Z - 3| = 5$ of Z-plane into a circle $\left|W + \dfrac{3}{16}\right| = \dfrac{5}{16}$ of W-plane.

4 (a) Answer the following questions briefly : **4**

(1) Define : Complex Series.

(2) Find Radius of convergence for the series $\sum_{n=1}^{\infty} n! Z^n$.

(3) Write expansion of SinhZ in Maclaurian series.

(4) State Maclaurian series of an analytic function $f(Z)$.

(b) Attempt any **one** : **2**

(1) Find region of convergence and radius of convergence of series $\sum_{1}^{\infty} \dfrac{Z^n}{3^n - 1}$.

(2) Expand $\dfrac{1}{1+Z}$ in Maclaurian's series.

(c) Attempt any **one** : **3**

(1) Prove that $e^z = e + e\sum_{n=1}^{\infty} \dfrac{(Z-1)^n}{n!}$.

(2) If $0 < |Z| < 4$ then prove that $\dfrac{1}{4Z - Z^2} = \sum_{n=0}^{\infty} \dfrac{Z^{n-1}}{4^{n+1}}$.

(d) Attempt any **one** : **5**

(1) State and prove Taylor's infinite series of an analytic function $f(Z)$.

(2) State and prove necessary and sufficient condition for complex sequence $\{Z_n\}$ to be convergent.

5 (a) Answer the following questions briefly : **4**

(1) Define : Residue of $f(z)$ at pole Z_0.

(2) Find Residue of $\frac{\cos Z}{Z}$ at $Z = 0$.

(3) Find Singular points of $\frac{\cos \pi Z}{(Z-1)(Z-2)}$.

(4) Which contour is used to integrate $\int_0^\infty \frac{dx}{1+x^2}$.

(b) Attempt any **one** : **2**

(1) Derive formula for finding residue of $f(z)$ at simple pole Z_0.

(2) Evaluate $\int_c \frac{5Z-2}{Z(Z-1)} dz$ where $C : |Z| = 2$.

(c) Attempt any one : **3**

(1) Evaluate $\int_c \frac{3Z^2+2}{(Z-1)(Z^2+9)} dZ$ where $C : |Z| = 2$.

(2) Find : Res $\left(\frac{1-e^Z}{Z^4}, 0\right)$.

(d) Attempt any **one** : **5**

(1) Using residue theorem prove that

$$\int_{-\infty}^{\infty} \frac{dx}{\left(1+x^2\right)^3} = \frac{3\pi}{8}$$

(2) State and prove Cauchy-Residue theorem.